LA-UR -77-2472

MASTER

CONF-771029--26

TITLE: PARAMETRIC STUDY OF PLASMA STARTUP SYSTEMS FOR TOKAMAKS

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SUBMITTED TO: The Seventh Symposium on Engineering Problems of Fusion Research, Hyatt Regency, Knoxville, TN, October 25-28, 1977.

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PARAMETRIC STUDY OF PLASMA STARTUP SYSTEMS FOR TOKAMAKS*

by

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Work performed under the auspices of USDOE.

Summary

Plasma initiation from a poloidal coil set is discussed for inducing the startup voltage where no magnetic flux is generated in the plasma region. A mathematical expression is reported which states the current distribution necessary in a toroidal shell for the zero field condition inside. The combination of the startup coil in one physical unit with the vertical field coil is considered. A flux plot is discussed for the case that the coil set is mounted outside the toroidal field coils and iron used for reducing the magnetic field energy.

Introduction

Motivated by the conclusions of Thomassen et al. (1976) that a large amount of energy is needed if the plasma is initiated by inserting a resistor ("blip" resistor) into the bias current carrying ohmic-heating coil, we made an effort to reduce the energy by a set of dedicated and unbiased blip coils. The blip coils are designed to generate zero field in the plasma region and supply the voltage pulse, whereas most of the blip energy is supplied by magnetic coupling from the VF coil circuits, thus requiring controlled power supplies to maintain the correct VF currents. We concluded, therefore, that the blip coils and their driver circuits may best be combined with those for the VF. The magnetic field calculations include the configuration with the combined VF-blip set located outside the TF coils so the assembly and maintenance of the tokamak will be facilitated. Calculations were started to include the effects of iron for reducing the poloidal field energy in a design with the external VF-blip coils.

Combination of Blip and EF Coils

It is possible to distribute any given current among the individual turns forming the vertical field coil so the plasma region is virtually excluded from the poloidal field generated by the current in the coil. Plasma initiation by inducing a pulsed voltage in the plasma region may thus be obtained by modulating the current in the equilibrium field coils.[a]

Our method of excluding the plasma region from the blip field is justified by the idealized case of a thin toroidal shell carrying current in the direction of only the major azimuth. In this case, the zero field condition, $B = 0$, is satisfied anywhere within the shell if the current distribution is given by

$$\frac{1}{r}\frac{dI}{d\theta} \quad B_\theta = \frac{I}{2\pi r}\left(1 - \frac{R}{r}\cos\theta\right) \tag{1}$$

where I is the total current in the shell, $dI/d\theta$ its poloidal distribution, r the minor radius of the shell, and θ a function of the poloidal, i.e., minor azimuth angle defined such that $\theta = 0$ on the shell's outside perimeter, which is located in the flux normal symmetry plane. A derivation of Eq. (1) is reported in the Appendix. Figure 1 shows a flux plot for a VF coil set with 10 discrete currents distributed such that in the intersection of the plasma region and the symmetry plane the ratio of the normalized average flux density over the total current, $2\pi < rP >/I = 2\%$. This corresponds to a flux density of approximately 20 G for a total blip current $I = 2$ MA.

The set of discrete currents was obtained from the computer program, TNSCOIL, by which the equation[1] for the axial component of the magnetic field of circular filaments is solved and the set of filament currents determined which satisfies, in the least squares sense, a given set of axial field gradient components under the following constraints:

a) The predetermined signs of the currents are to remain invariant.

b) Several (≤ 8) filaments may be ganged so their currents are identical and thus approximate the conductor of finite size.

c) Some filaments may be constrained to carry currents, invariant under the least squares fitting process.

The least squares fitting routine uses the Householder transformation which multiplies the coefficient matrix so a triangular matrix is obtained. This algorithm has the important properties of numerical stability, high accuracy even for large arrays, and fast execution for the case of the fully ranked coefficient matrix that may be overdetermined, exactly determined, or underdetermined.

The VF coil in Fig. 1 is located outside the TF coils to facilitate the assembly and maintenance of the tokamak. The electrical circuit with blip flux obtained by modulation of the vertical field currents requires a programmed power supply to maintain the fixed current balance among the turns of the VF as well as the blip field for any value of the total current. This can be done by a linear feedback circuit where the total VF current is controlled to image the plasma current and the total blip current follows from the electromagnetic circuit including the blip voltage source.

Effects of Iron

The great advantage from a practical point of view to have the VF-and-blip coils mounted outside the TF coils is bought at the expense of a large increase in field energy, unless effective use is made of iron to reduce the reluctance of the poloidal flux path. This concept is illustrated by the flux plot of Fig. 2 in which some of the effects of the following design features are illustrated:

a) In addition to providing the VF and blip field, the VF coils are used as drivers for reversing the flux corresponding to −2 T at the start and +2 T at the end of the operating pulse.

b) A constant current superconducting bias coil is wound on the center post iron to provide the bias field of −2 T.

Analysis and operation are complicated by the iron's presence at varying levels of saturation because the inductance matrix varies significantly over the operating

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[a]Superposition of the vertical field (VF) and blip field requires linearity in the B versus H relationship as a boundary condition.

range. In the TNS case, it is expected that a flux swing of 40 Vs over the range -2 T···2 T may be obtained without varying the plasma major or minor radii.

For computing the inductance matrix, program TNSCOIL can be used after the addition of an algorithm has been implemented for computing the flux in air in the presence of its image in the iron space and the flux in iron in the presence of its image in air. For solving the operational problem, a pickup loop placed around the centerpost iron may be used in conjunction with an integrator to supply flux density information for on-line correction of the control circuit's output for the VF coil current control.

Acknowledgment

Many useful suggestions were gathered from discussions with M. Peng, D. Weldon, and P. Marcus.

References

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H. F. Vogel, K. I. Thomassen, W. Bird, and F. M. Heck, "Pulsed Energy and Switching Requirements for Tokamak Ohmic Heating," Proc. IEEE Internat. Pulsed Power Conf., Nov. 9-11, 1976, Lubbock, TX, p. III 29-1.

2. William R. Smythe, "Static and Dynamic Electricity McGraw-Hill Book, New York, 1968, p. 290.

Appendix

Conditions for Zero Field in the Current Conducting Toroidal Shell

Consider a toroidal shell with thin wall of minor radius r and major radius R. We want to know the current distribution in the wall that generates zero flux inside the shell.

The zero field condition inside the thin shell requires that the shell surface be one with constant potential vector $\vec{A}$. It is useful, therefore, to use the toroidal coordinates obtained by twirling the circles shown in Fig. A-1 about the z-axis. The toroidal coordinates satisfy the necessary conditions of orthogonality and separability of the Laplace equation into ordinary differential equations. The coordinates are defined by

$$\begin{aligned} x &= a \sinh\eta \cos\psi/(\cosh\eta - \cos\theta) \\ y &= a \sinh\eta \sin\psi/(\cosh\eta - \cos\theta) \qquad \text{(A-1)} \\ z &= a \sin\theta/(\cosh\eta - \cos\theta) \quad . \end{aligned}$$

The surfaces of constant η are toroids, those of constant θ are spherical bowls, and the surfaces, ψ = const, are half planes through the axis of symmetry (Fig. A-1). The equations for these surfaces are easily obtained from Eq. (A-1), so are some useful relationships for the η = const and a - values of the shell with given minor and major radii r and R; i.e.,

$$a = r[(R/r)^2 - 1]^{1/2} \qquad \text{(A-2)}$$

$$\eta = \text{arc cosh } (R/r) \qquad \text{(A-3)}$$

The curl of the vector potential in toroidal coordinates is obtained in the usual way of the general expression for the curl and the metric coefficients for the coordinates considered, i.e., $g_{11} = (\partial x/\partial\eta)^2 + (\partial y/\partial\eta)^2 + (\partial z/\partial\eta)^2$, $g_{22} = (\partial x/\partial\theta)^2 + (\partial y/\partial\theta)^2 + (\partial z/\partial\psi)^2$, etc., which yield the matrix

$$(g_{ij}) = \frac{a^2}{(\cosh\eta - \cos\theta)^2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sinh^2\eta & 0 \\ 0 & 0 & a\sinh\eta \end{bmatrix} \qquad \text{(A-4)}$$

The off diagonal elements are zero, thus indicating orthogonality. We obtain, thus, an expression for the curl, i.e.,

$$\vec{\nabla} \times \vec{A} = \frac{(\cosh\eta - \cos\theta)^2}{a \sinh\eta} \begin{bmatrix} \vec{a}_\eta & \vec{a}_\theta & \vec{a}_\psi \\ \partial/\partial\eta & \partial/\partial\theta & \partial/\partial\psi \\ \frac{A_\eta}{(\cosh\eta - \cos\theta)} & \frac{A_\theta}{(\cosh\eta - \cos\theta)} & \frac{A_\psi \sinh\eta}{(\cosh\eta - \cos\theta)} \end{bmatrix} \qquad \text{(A-5)}$$

We define one flux surface (η = const, $A = A_\psi$ = const) at the shell where the ψ directional current is imposed and where, therefore, $A_\eta = A_\theta = \partial A/\partial\theta = \partial A/\partial\psi = 0$.

These conditions follow from the symmetry of the defined flux surface. For this one dimensional case, we get

$$B_\theta = \vec{\nabla} \times \vec{A} = A \frac{1 - \cosh\eta \cos\theta}{a \sinh\eta} \qquad \text{(A-6)}$$

A point should be made regarding the general integral of Laplace's equation in this coordinate system.[1] The separation equations lead, for the case of axial symmetry, i.e., ψ-independence of the potential, ϕ, to

$$\begin{aligned} \phi &= (\cosh\eta - \cos\theta)^{1/2} P_{n-1/2}(\cosh\eta) \begin{pmatrix} \sin \\ \cos \end{pmatrix} n\theta \\ \phi &= (\cosh\eta - \cos\theta)^{1/2} Q_{n-1/2}(\cosh\eta) \begin{pmatrix} \sin \\ \cos \end{pmatrix} n\theta \end{aligned} \qquad \text{(A-7)}^{a}$$

Equation (A-7) shows dependence of η and θ, and the one dimensional case in which the potential depends on η only or θ only cannot occur in these coordinates. This is because the separation function $R = R(\eta, \theta, \psi)$ in the potential function $\phi = U_\eta(\eta) \cdot U_\theta(\theta) \cdot U_\psi(\psi)/R$ is given by

$$R = (\cosh\eta - \cos\theta)^{-1/2} \ , \qquad \text{(A-8)}^{b}$$

which is a function of η and θ and makes a one dimensional solution impossible. The surfaces η = const other than the one defined on the shell are, therefore, not equipotentials, even in the scalar potential case of the electrically charged metal toroid.

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a) $P_{n-1/2}(\mu)$ and $Q_{n-1/2}(\mu)$ are the associated Legendre functions.

b) $R = R(\eta, \theta, \psi)$ is not related to the similar notation used elsewhere in this paper for the major radius.

By substituting Eqs. (A-2) and (A-3) in Eq. (A-6) and eliminating the vector potential from the boundary condition that the total current equal I, we get the surface gradient

$$H \equiv H_\theta \doteq \frac{I}{2\pi r}\left(1 - \frac{R}{r}\cos\theta\right) \qquad \text{(A-9)}$$

for a, η real, positive according to Eqs. (A-2) and (A-3). The angle θ is obtained from Eq. (A-1) for given $a = (R^2 - r^2)^{1/2}$ and η = arc cosh (R/r).

References

A-1. Parry Moon and Domina Eberle Spencer, "Field Theory for Engineers," D. Van Nostrand, Princeton, 1961.

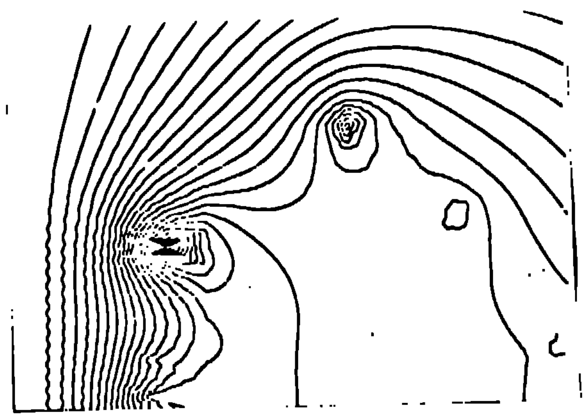

Fig. 1. Blip field for coils outside TF coils.

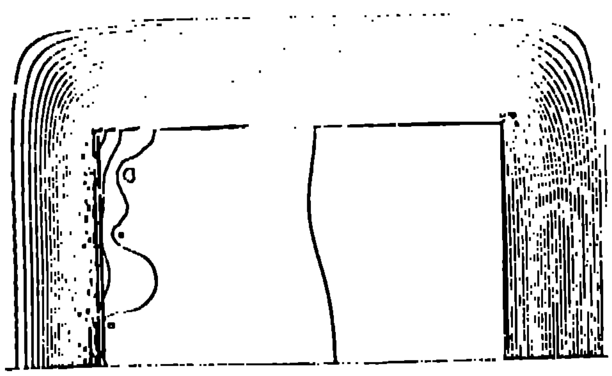

Fig. 2. Blip field for coils outside TF coils and with bias winding generating 2 T in iron core.

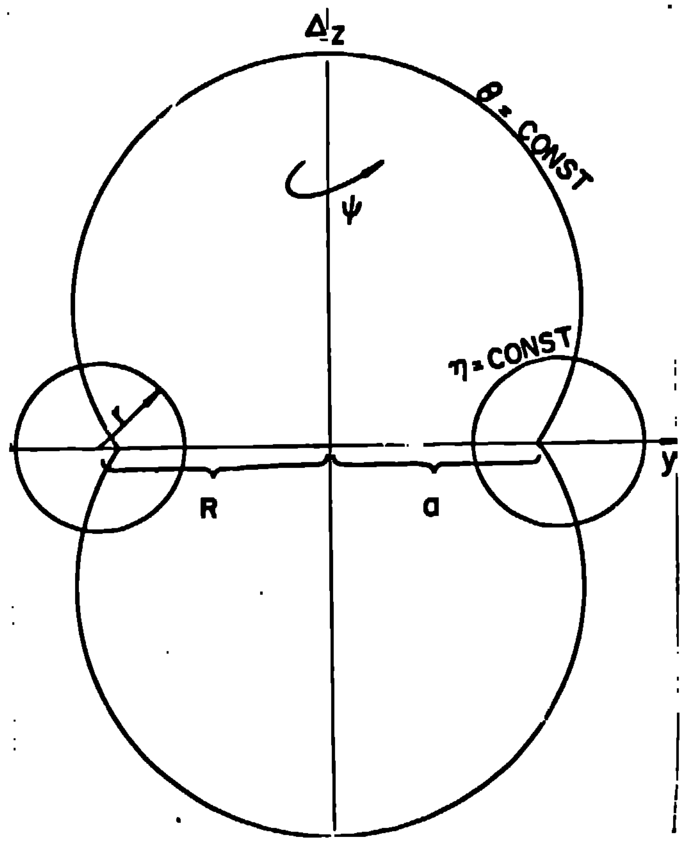

Fig. A-1. Toroidal coordinate system.